Virtuális erők és virtuális elmozdulások

A virtuális elmozdulásrendszer egy tetszőleges, geometriailag lehetséges elmozdulásrendszer változatlan geometriai feltételek mellett képzett differenciálisan kicsiny megváltozása, variációja.

A virtuális elmozdulások tétele:

A virtuális elmozdulások tétele szerint egy statikailag lehetséges erőrendszer bármely virtuális elmozdulásrendszeren végzett munkája zérus.

$$\delta W = \delta W_k + \delta W_b = 0$$

(Az egyensúlyi feltételt fejezi ki.)

A virtuális erőrendszer egy tetszőleges statikailag lehetséges erőrendszer, változatlan statikai kerületi feltételek mellett képzett differenciálisan kicsiny megváltozása, variációja.

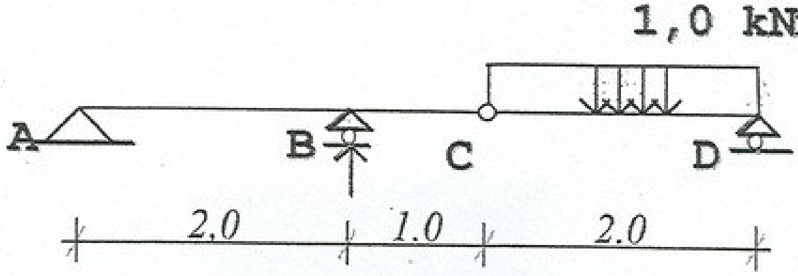
A virtuális erők tétele:

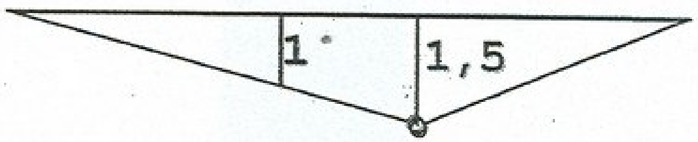
A virtuális erők tétele szerint egy geometriailag lehetséges elmozdulásrendszernek bármely virtuális erőrendszeren végzett (kiegészítő) munkája zérus.

$$\delta \tilde{W} = \delta \tilde{W}_k + \delta \tilde{W}_b = 0$$

(Ez az elmozdulások és az alakváltozások kompatibilitását fejezi ki.)

1. Virtuális elmozdulás segítségével határozzuk meg a „B” reakció értékét!





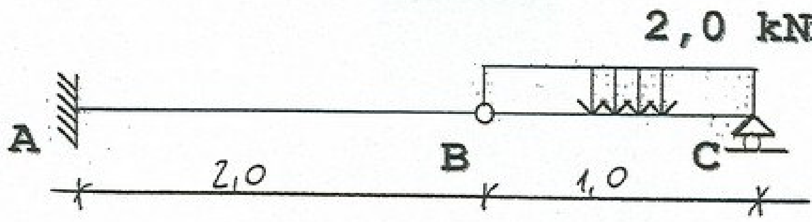
$$-B\cdot 1.0+2\cdot 1.5\cdot\frac{1}{2}=0$$

$\underline{B=1.5kN}(\uparrow)$

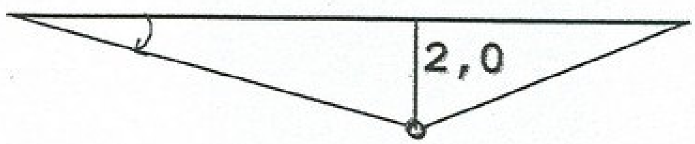
$\delta L_k=0$

(nincs alakváltozás = nincs belső munka)

2. Határozzuk meg a befogási nyomaték értékét virtuális elfordulás segítségével!



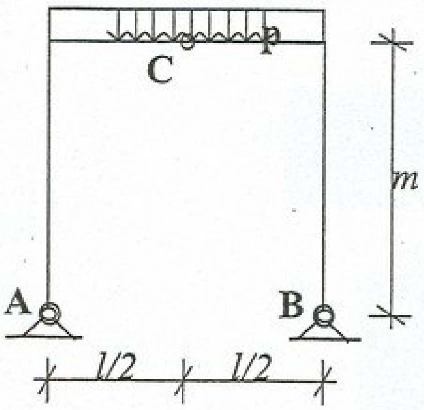
$\varphi_A=1,0$

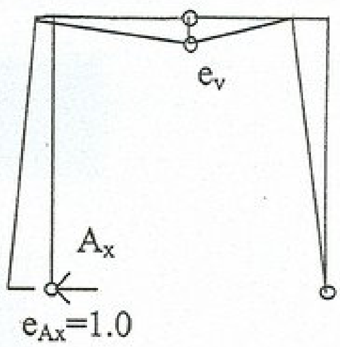


$$1\cdot M_A+\frac{1\cdot 2.0}{2.0}\cdot 2.0=0$$

$\underline{M_A=-2.0\cdot kNm}$

3. Határozzuk meg az A_x reakciót virtuális elmozdulás segítségével!





$$\vartheta_c = \frac{1}{m} \rightarrow \varphi_B = \frac{1}{2m}$$

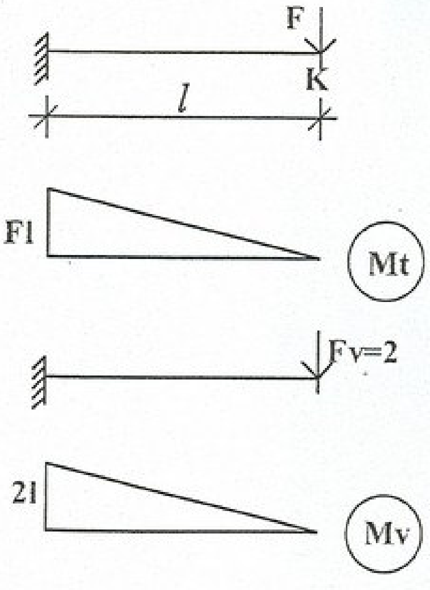
$$\rightarrow e_C = \frac{l}{4\cdot m}(\downarrow)$$

$$A_x \cdot 1 + \frac{l\cdot l\cdot p}{4\cdot m\cdot 2} = 0$$

$$\underline{A_x = -\frac{p\cdot l^2}{8\cdot m}(\rightarrow)}$$

1. Virtuális er felhasználásával számítsa ki e_{ky}-t!

EI= állandó



$$\begin{pmatrix} \delta\tilde{L}_K = \int \underline{e}^* \cdot \delta \underline{q} \cdot dA \\ \delta\tilde{L}_B = -\int \underline{\underline{\varepsilon}}^* \cdot \delta \underline{\underline{\sigma}} \cdot dV \end{pmatrix}$$

$$\delta\tilde{L}_K = F_v \cdot e_{ky}$$

$$\delta\tilde{L}_{B} = -\int M_v \cdot \kappa \cdot ds \left(-\int N_v \cdot \Delta l \cdot ds\right)$$

$$\kappa = \frac{M_v}{E\cdot I}$$

$$\Delta l = \frac{N_v}{E\cdot A}$$

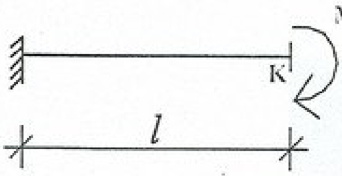
$$\int \underline{e}^* \cdot \delta \underline{q} \cdot dA - \int \underline{\underline{\varepsilon}}^* \cdot \delta \underline{\underline{\sigma}} \cdot dV = 0$$

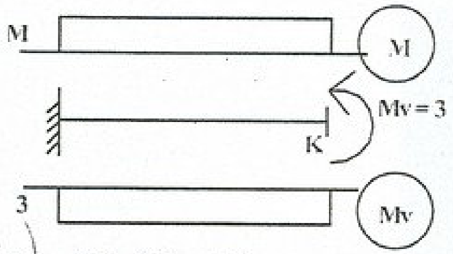
$$2\cdot e_{ky} = \int_l \frac{M_v \cdot M_t}{E\cdot I} \cdot ds$$

$$2\cdot e_{ky} = \frac{F\cdot l\cdot l}{2} \cdot \frac{2}{3} \cdot 2\cdot l \cdot \frac{1}{E\cdot I}$$

$$e_{ky} = \frac{F\cdot l^3}{3\cdot E\cdot I}$$

2. EI= állandó $\varphi_K = ?$

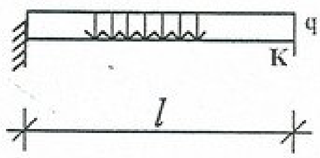


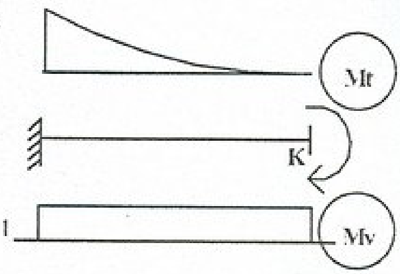


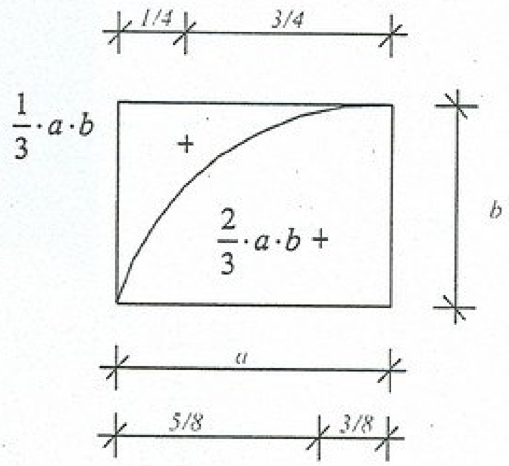
$$3 \cdot \varphi_K = -\frac{1}{E \cdot I} \cdot M \cdot l \cdot 3$$

$$\varphi_K = -\frac{M \cdot l}{E \cdot I}$$

3. EI= állandó $\varphi_K = ?$





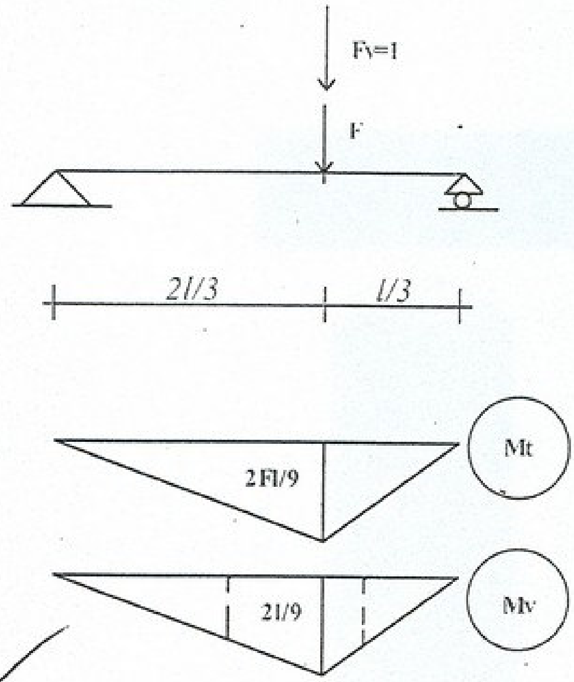


A magasabb fokszámú alatti területet kell venni.

$$\varphi_K \cdot 1 = \frac{1}{3} \cdot \frac{\frac{q \cdot l^2}{2} \cdot l}{E \cdot I} \cdot 1$$

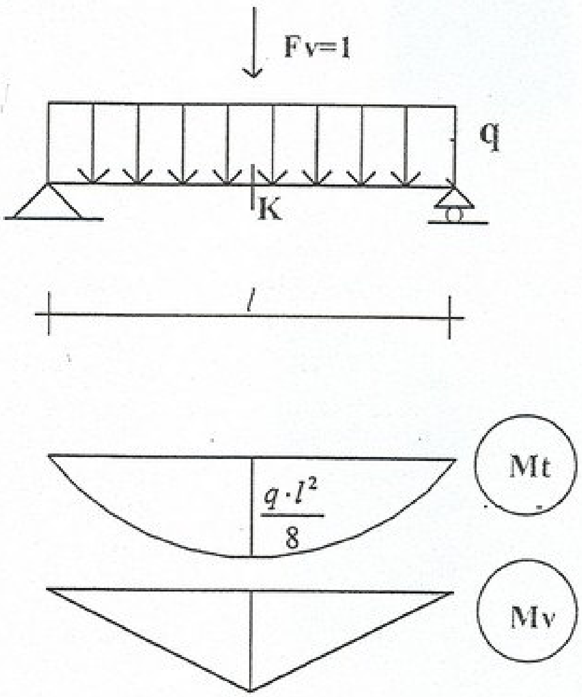
$$\varphi_K = \frac{q \cdot l^3}{6 \cdot E \cdot I}$$

4. EI= állandó e_{Ky}=?



$$1\cdot e_{Ky}=\frac{2\cdot F\cdot l}{9}\cdot\frac{2\cdot l}{3}\cdot\frac{1}{2\cdot E\cdot I}\cdot\frac{2}{3}\cdot\frac{2\cdot l}{9}+$$
$$+\frac{2\cdot F\cdot l}{9}\cdot\frac{l}{3}\cdot\frac{1}{2\cdot E\cdot I}\cdot\frac{2}{3}\cdot\frac{2\cdot l}{9}$$
$$e_{Ky}=\frac{4\cdot F\cdot l^3}{243\cdot E\cdot I}$$

5. EI= állandó e_{Ky}=?



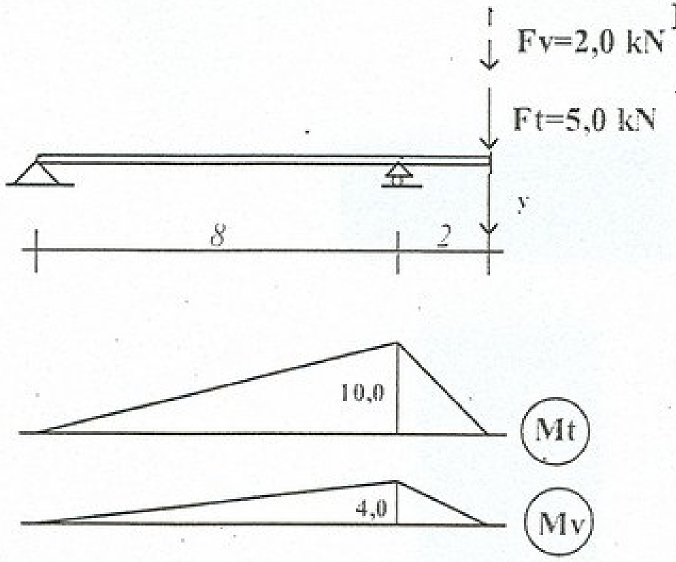
$$\frac{q\cdot l^2}{8}$$

$$\frac{F\cdot l}{4}=\frac{l}{4}$$

$$1\cdot e_{Ky}=2\cdot\left[\left(\frac{2}{3}\cdot\frac{q\cdot l^2}{8}\right)\cdot\frac{l}{2}\cdot\frac{5}{8}\cdot\frac{l}{4}\right]\cdot\frac{1}{E\cdot I}$$
$$e_{Ky}=\frac{5\cdot q\cdot l^4}{384\cdot E\cdot I}$$

6. Virtuális er felhasználásával számítsa ki e_{ky} értékét!

EI= állandó= 1000 kN*m²



$$F_v \cdot e_{Ky} = \int_l \frac{M_v \cdot M_t}{E \cdot I} \cdot dl$$

$$2 \cdot e_{Ky} = \frac{1}{1000} \cdot \left[\frac{8 \cdot 10}{2} \cdot 4 \cdot \frac{2}{3} + \frac{2 \cdot 10}{2} \cdot 4 \cdot \frac{2}{3} \right]$$

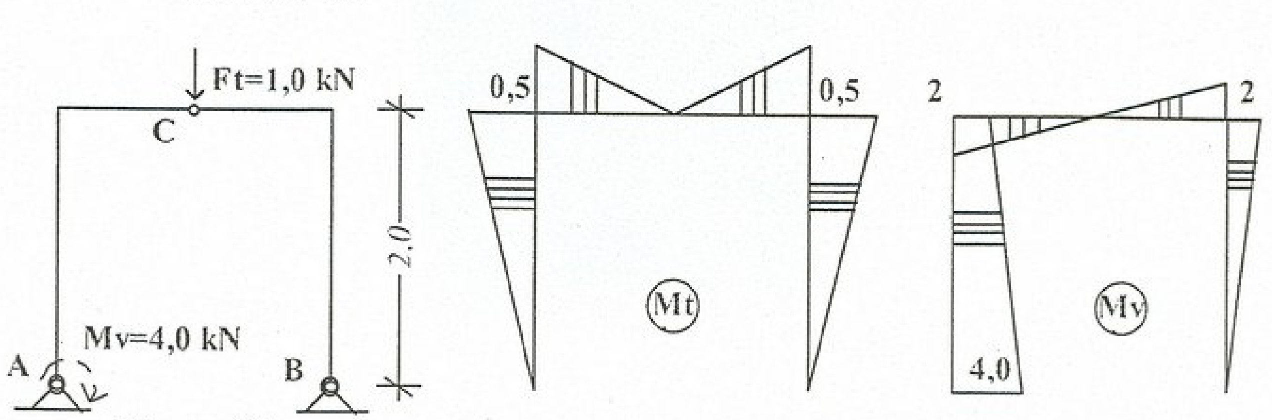
$$e_{Ky} = \frac{66,67}{1000} = 0,066 \cdot = 66mm$$

$$e_{Ky} = \frac{2}{30} = 0,067m(\downarrow)$$

Ellen rizzük a kis elmozdulások számítási eljárásának megfelel en!

7. Virtuális nyomaték felhasználásával számítsa ki φ_A értékét!

EI= 1000 kN*m²



$$4\cdot\varphi_A=\int_l \frac{M_v\cdot M_t}{E\cdot I}\cdot dl=\left[\frac{2\cdot 0{,}5}{2}\cdot 2\cdot\frac{2}{3}+\frac{1\cdot 0{,}5}{2}\cdot 2\cdot\frac{2}{3}-\frac{1\cdot 0{,}5}{2}\cdot 2\cdot\frac{2}{3}-\frac{2\cdot 0{,}5}{2}\cdot\left(2+2\cdot\frac{1}{3}\right)\right]\cdot\frac{1}{1000}=-\frac{1}{6000}$$

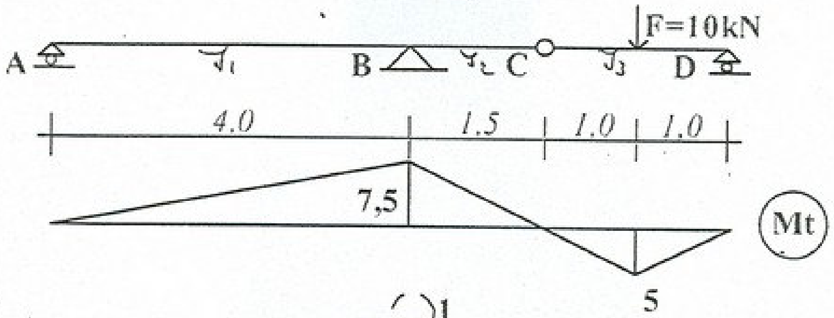
φ_A= -0.000167

Ellen rizzük a kis elmozdulások elméletével!

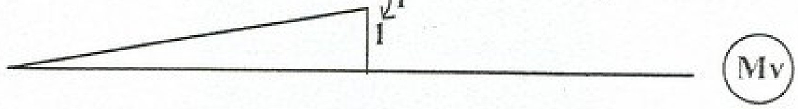
(Részeredmény: φ_C=7/6 (nagyított!))

8. a. φ_B=?
b. ϑ_C=?

EI_1=2000
EI_2=2200
EI_3=2300

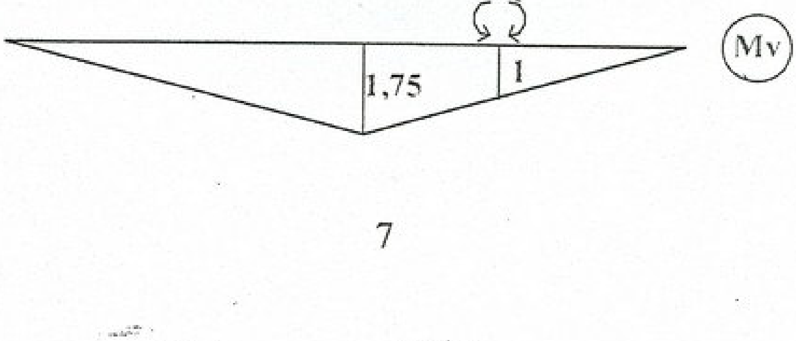


a.



$$\varphi_B=\frac{4\cdot 7{,}5}{2}\cdot\frac{2}{3}\cdot\frac{1}{2000}=0{,}005\,rad$$

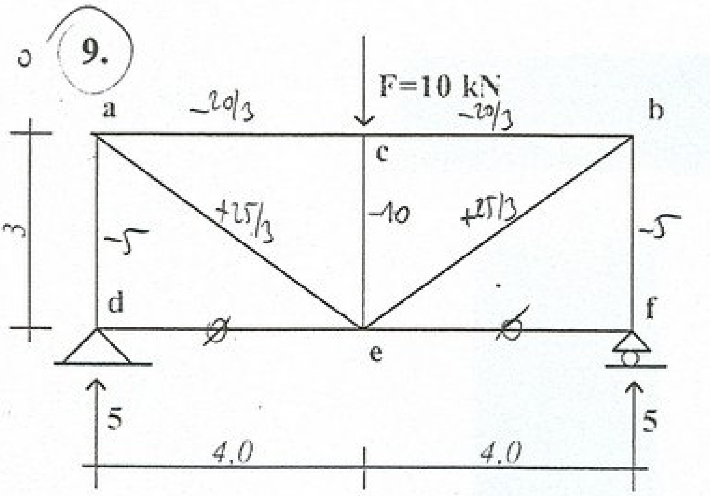
b.



$$\vartheta_C = -\frac{4\cdot 7{,}5}{2}\cdot\frac{2}{3}\cdot 1{,}75\cdot\frac{1}{2000} - \frac{7{,}5\cdot 1{,}5}{2}\cdot\left(1+\frac{2}{3}\cdot 0{,}75\right)\cdot\frac{1}{2200} + \left[\frac{5\cdot 1}{2}\cdot\left(0{,}5+0{,}5\cdot\frac{1}{3}\right)+\frac{5\cdot 1}{2}\frac{2}{3}\cdot 0{,}5\right]\cdot\frac{1}{2300}$$

$$\vartheta_C = 0{,}0115 rad$$

9.



EA_I=200 kN

Δu_{ab}=?

$$\int \frac{N_v\cdot N_t}{E\cdot A}\cdot dl$$

$$S_{ac} = S_{bc} = \frac{20}{3} kN(-)$$

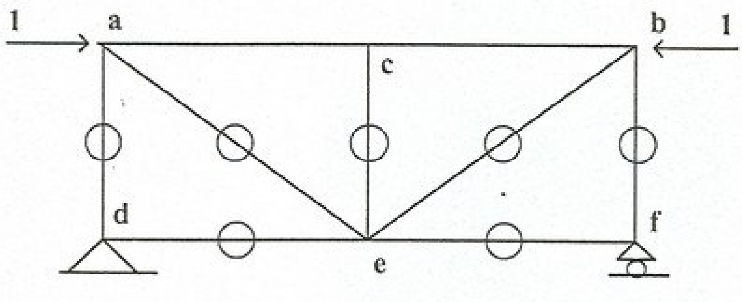
$$S_{ce} = 10kN(-)$$

$$S_{ef} = 0kN$$

$$S_{bf} = 5kN(-)$$

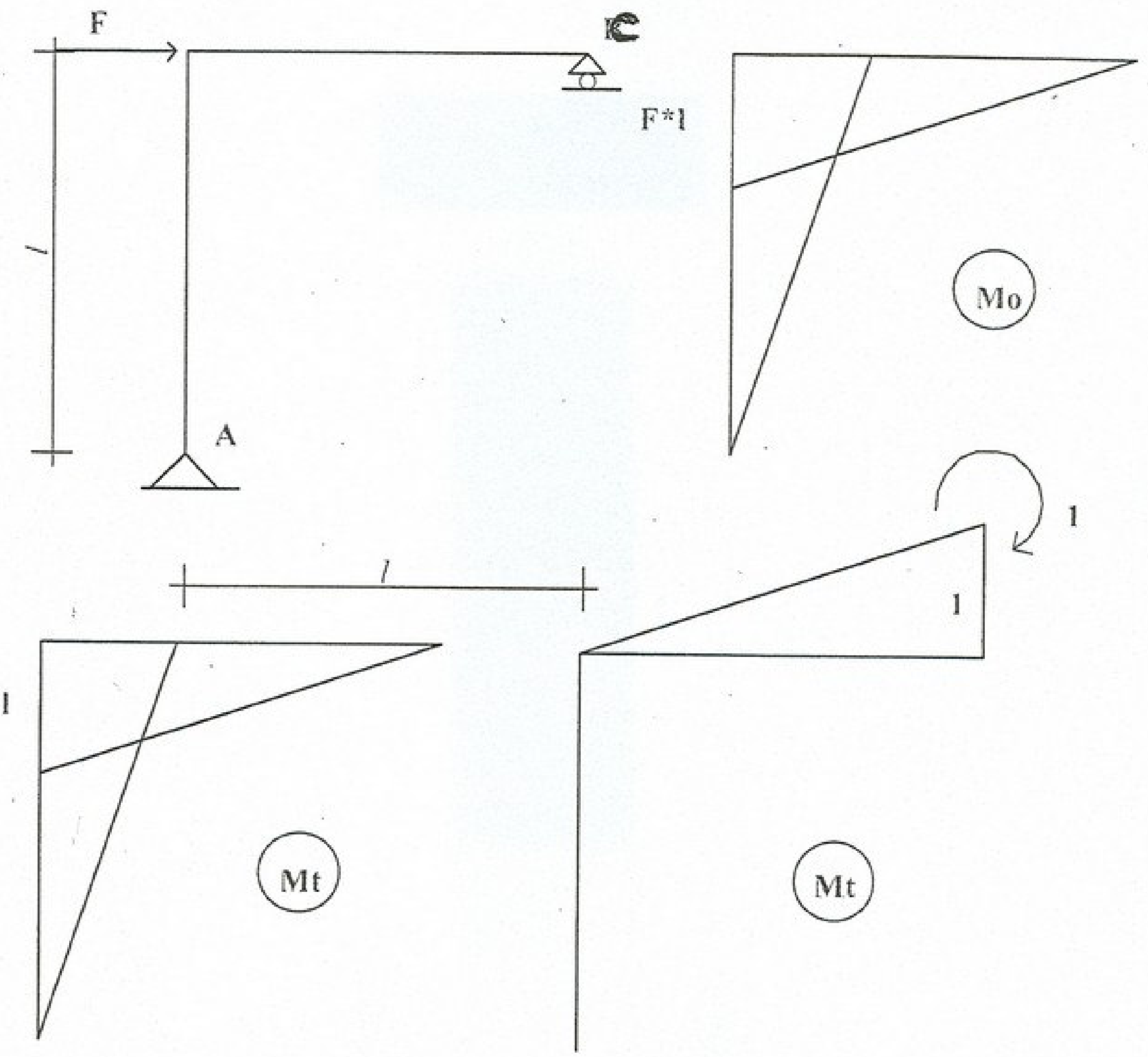
$$S_{ad} = 5kN(-)$$

$$S_{ae} = \frac{25}{3} kN(+)$$



$$\Delta u_{ab} = 2\cdot\frac{20}{3}\cdot 1\cdot\frac{1}{200}\cdot 4 = \underline{\underline{0{,}266m}}$$

10. EI= const. e_{cx}=?, φ_{cx}=?

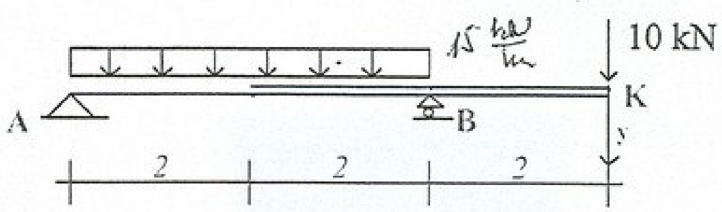


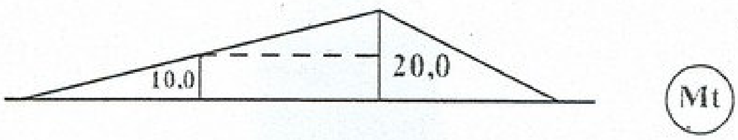
$$e_{Cx} = \frac{2\cdot F\cdot l\cdot l}{2}\cdot\frac{2}{3}\cdot l\cdot\frac{1}{E\cdot I} = \frac{2\cdot F\cdot l^3}{3\cdot E\cdot I}$$

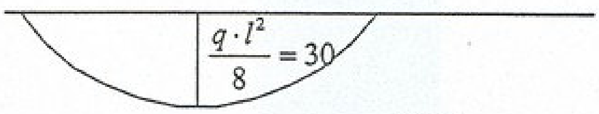
$$\varphi_{Cx} = \frac{-F\cdot l\cdot l}{2}\cdot\frac{1}{3}\cdot\frac{1}{E\cdot I} = \frac{-F\cdot l^2}{6\cdot E\cdot I}$$

11. EI_1=1000 kNm^2 $\qquad\qquad$ φ_B=?

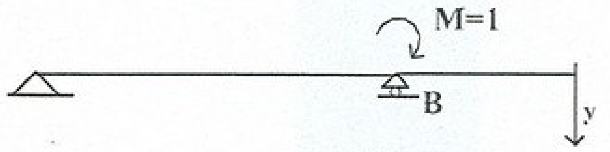
EI_2=2000 kNm^2 $\qquad\qquad$ e_{ky}=?

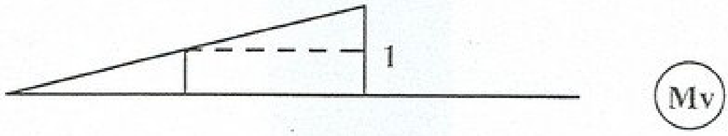






a.



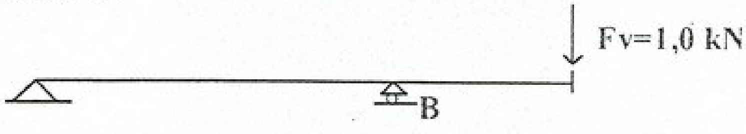


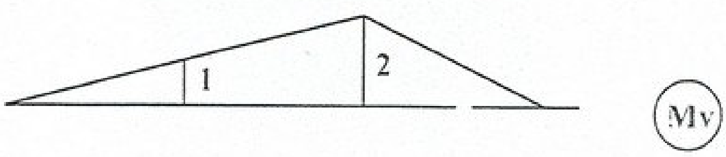
$$1\cdot\varphi_B=\frac{1}{1000}\cdot\left[\frac{2\cdot10}{2}\cdot\frac{2}{3}\cdot\frac{1}{2}+\frac{10\cdot2}{2}\cdot0.75+\frac{10\cdot2}{2}\cdot\frac{1}{2}\cdot\left(\frac{1}{2}+\frac{1}{2}\cdot\frac{2}{3}\right)-\frac{2}{3}\cdot2\cdot30\cdot\frac{5}{8}\cdot\frac{1}{2}-\frac{2}{3}\cdot2\cdot30\cdot\frac{1}{2}\cdot\left(\frac{1}{2}+\frac{3}{8}\cdot\frac{1}{2}\right)\right]$$

$$\varphi_B=-\frac{11.25}{1000}$$

φ_B=-0.01125 rad

b.





$$1\cdot e_{ky}=\frac{1}{1000}\left[\frac{2\cdot 10}{2}\cdot\frac{2}{3}\cdot 1+\frac{2\cdot 10}{2}\cdot 1.5+\frac{2\cdot 10}{2}\cdot\frac{1}{2}\cdot\left(1+\frac{2}{3}\right)+\frac{2\cdot 20}{2}\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot 2\right]-$$

$$-\frac{1}{1000}\left[\frac{2}{3}\cdot 2\cdot 30\cdot\frac{5}{8}\cdot 1+\frac{2}{3}\cdot 2\cdot 30\cdot\left(1+\frac{3}{8}\right)\right]$$

e_{ky}=-0.00917 m (↑)

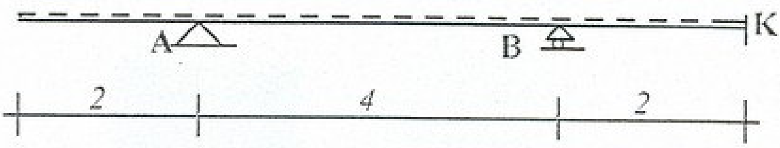
15.

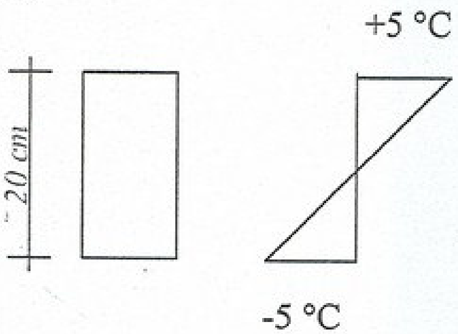
h=20cm

$\alpha=1.2\bullet 10^{-5}$ 1/°C

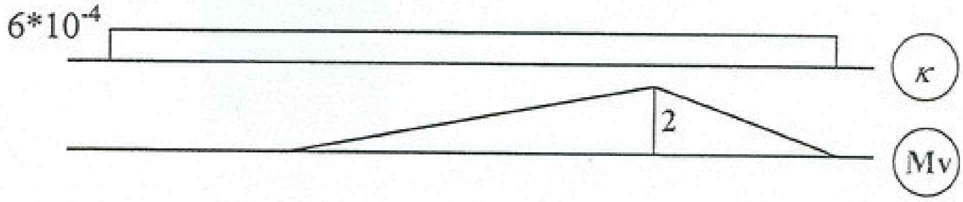
EI=1000 kNm^2

e_{ky}=?





$$\kappa=\frac{\alpha\cdot\Delta t}{h}=\frac{1.2\cdot 10^{-5}\cdot 10}{0.2}=6\cdot 10^{-4}$$

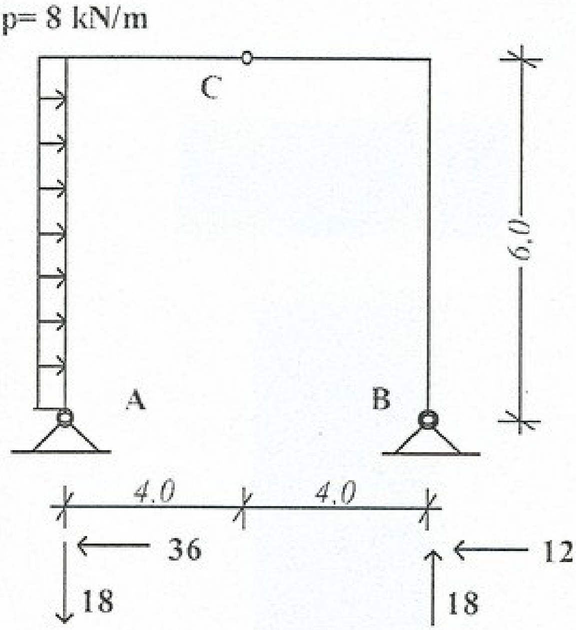


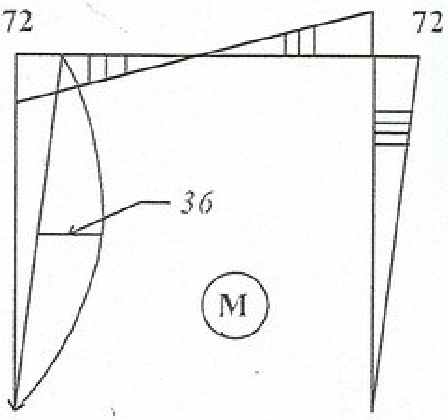
$$e_{ky}=\frac{4\cdot 2}{2}\cdot 6\cdot 10^{-4}+\frac{2\cdot 2}{2}\cdot 6\cdot 10^{-4}$$

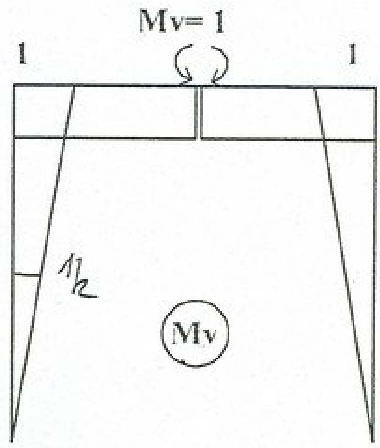
e_{ky}=3.6*10^{-3} m (↓)

16. EI= const.=10^5 kNm^2

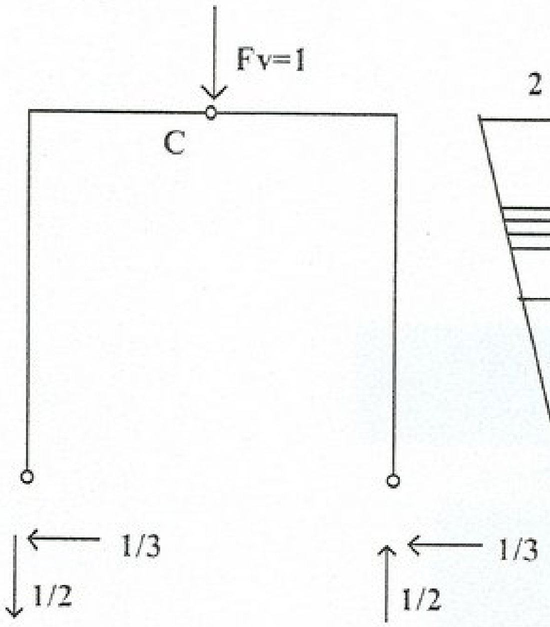
ϑ_C=?, e_{cy}=?

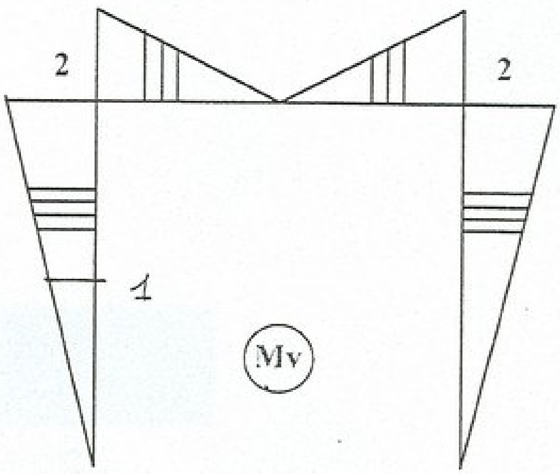






$$\vartheta_c = \frac{1}{10^5} \cdot \frac{2}{3} \cdot 6 \cdot 36 \cdot \frac{1}{2} = 7{,}2 \cdot 10^{-4}$$





$$e_{cy} = -\frac{1}{10^5}\cdot\frac{2}{3}\cdot 6\cdot 36\cdot 1$$

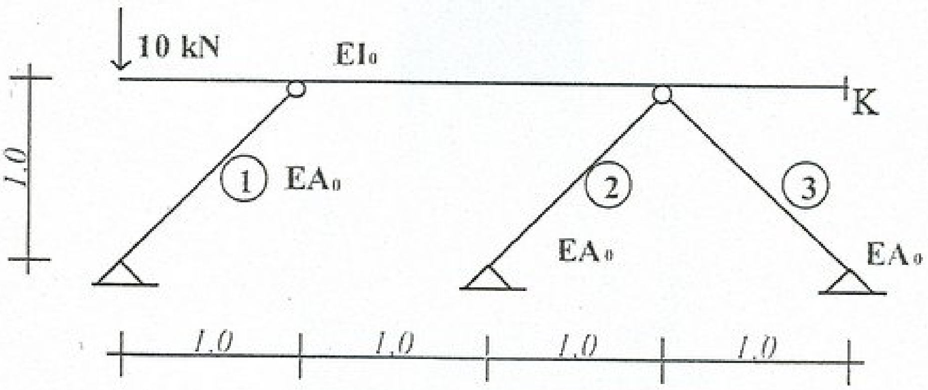
$e_{cy}=-1{,}44*10^{-3}$ m (↑)

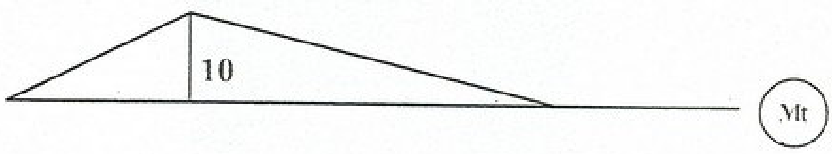
17. $EI_0=10^4$ kNm^2, $EA_g \to \infty$

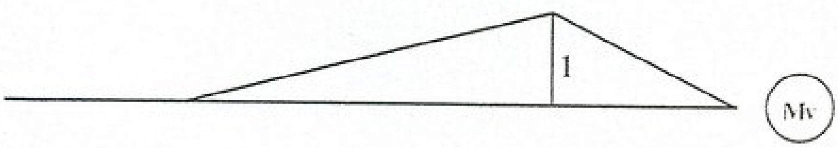
~~$EA_0=2\cdot10^4$ kN~~ $2\cdot10^4$ kN

$i_0^2=0{,}5$ m^2

$e_{ky}=?$







$S_{10} = 15 \cdot \sqrt{2} kN(-)$ $\qquad S_{11} = \frac{\sqrt{2}}{2} kN(+)$

$S_{20} = 10 \cdot \sqrt{2} kN(+)$ $\qquad S_{21} = -\sqrt{2} kN(-)$

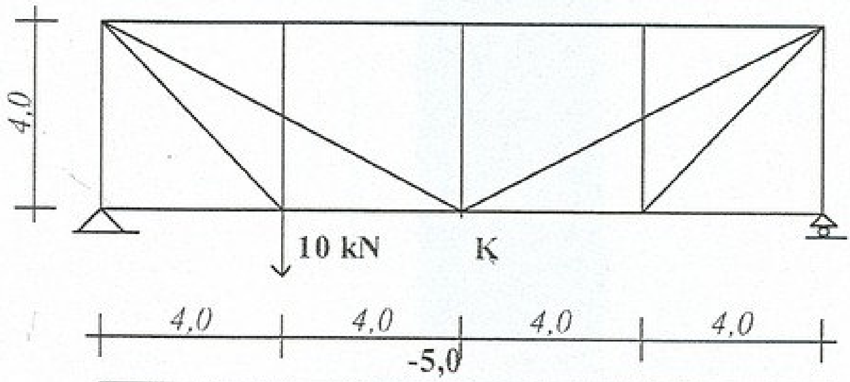
$S_{30} = 5 \cdot \sqrt{2} kN(-)$ $\qquad S_{31} = \frac{\sqrt{2}}{2} kN(-)$

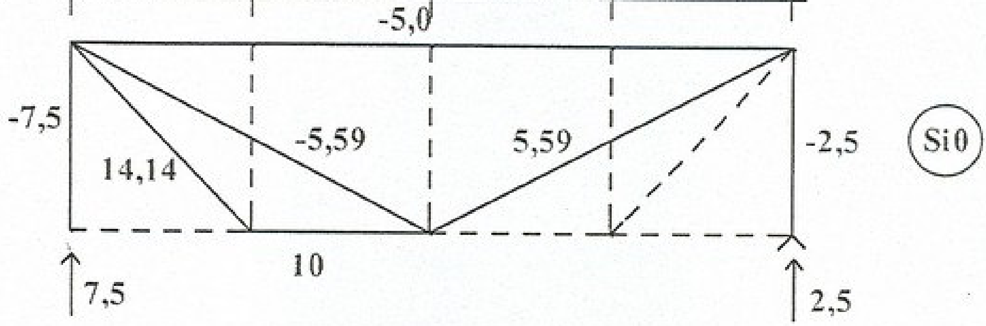
$$e_{Ky} = 10^{-4}\left[\frac{10 \cdot 2}{2} \cdot \frac{1}{3} + 0{,}5 \cdot \sqrt{2} \cdot \left(-15 \cdot \sqrt{2} \cdot \frac{\sqrt{2}}{2} - 10 \cdot \sqrt{2} \cdot \sqrt{2} + 5 \cdot \sqrt{2} \cdot \frac{\sqrt{2}}{2}\right)\right]$$

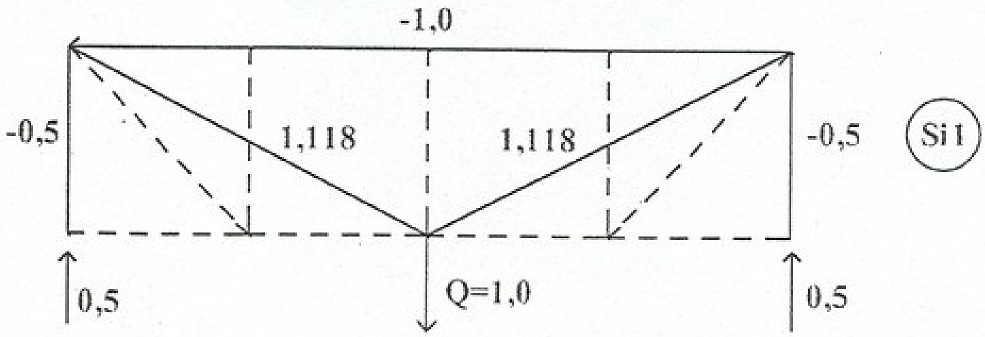
$$e_{Ky} = 17{,}88 \cdot 10^{-4} m$$

18. EA= állandó

e_{Ky}=?







$$e_{Ky} = \sum \frac{S_{io} \cdot S_{i1}}{E \cdot A_i} \cdot s_i$$

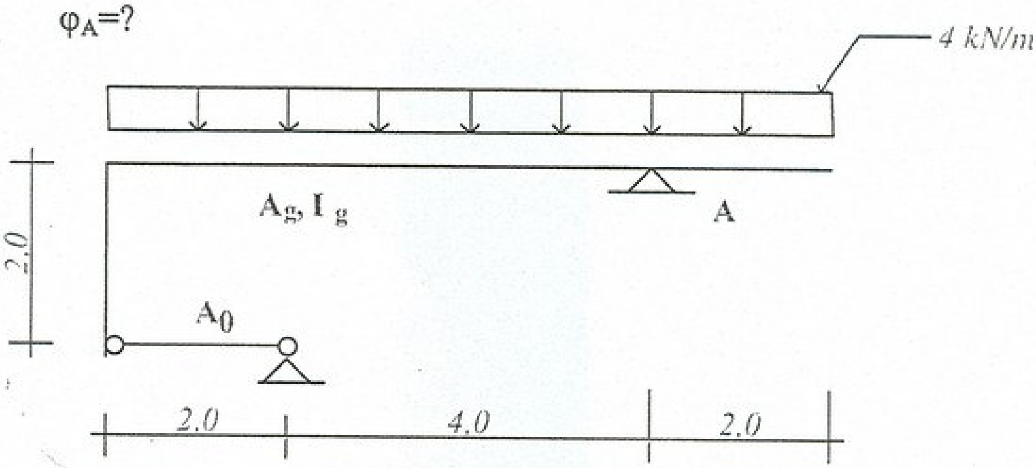
$$E \cdot A \cdot e_{Ky} = (e_{Ky}) = (1{,}5 \cdot 16 + 0{,}5 \cdot 7{,}5 \cdot 4 + 0{,}5 \cdot 2{,}5 \cdot 4) = 100 kNm$$

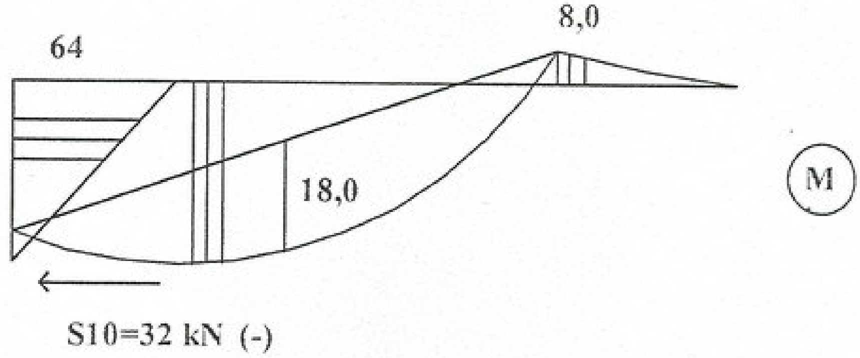
19. $I_g = I_0$

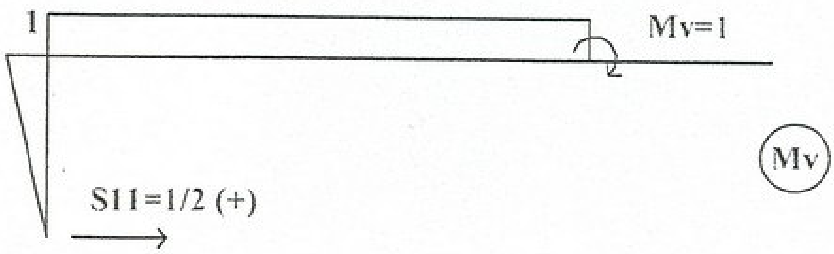
$A_g/A_0 = \lambda_g = 3{,}0$

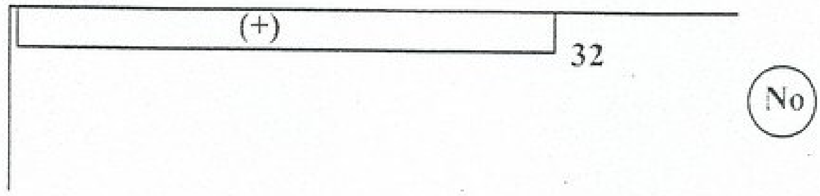
$i_0^2 = 2{,}0 m^2$

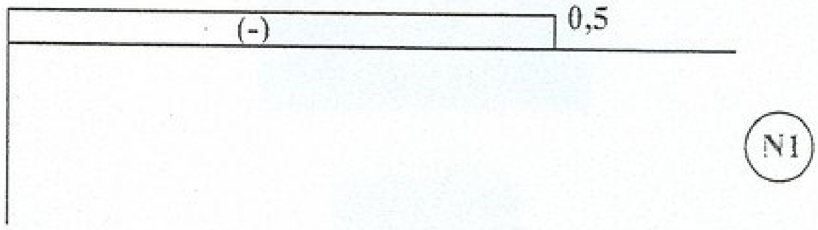
$\varphi_A = ?$











M-b 1:

$$(\varphi_A)' = \frac{8}{2}\cdot 6\cdot 1 - \frac{64}{2}\cdot 6\cdot 1 - \frac{18\cdot 6\cdot 2}{3} - \frac{64\cdot 2}{2}\cdot\frac{2}{3} = -282{,}66kNm^2$$

N-b 1:

$$(\varphi_A)'' = 2{,}0\cdot\left(-32\cdot 6\cdot\frac{1}{2}\cdot\frac{1}{3} - \frac{32}{2}\cdot 2\right) = -128{,}0kNm^2$$

$$(\varphi_A) = (\varphi_A)' + (\varphi_A)'' = \underline{\underline{-410{,}66kNm^2}}$$